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Evolution of the Perturbations in Baryon-Photon Coupled Plasma:

We now have a more precise treatment of the evolution of perturbations in a plasma of baryons and photons.

As we saw last time, dark matter decouples early on, and hence its perturbations can grow as a result of gravitational attraction freely. Before matter domination, however, the growth is very mild and insignificant.

At $t > t_{EQ}$ ($\sim 100{,}000$ yr) we have:

$$\ddot{\delta}_{DM} + \frac{4}{3t}\dot{\delta}_{DM} - \frac{2}{3t^2}\delta_{DM} = 0$$

Here we have used the fact that $H = \frac{2}{3t}$ for a matter-dominated universe, and have considered a single component (i.e. dark matter) fluid for simplicity.

The above equation has a growing and a decaying solution, denoted by $\delta_+(t)$ and $\delta_-(t)$ respectively:

$$\boxed{\delta_+(t) = \delta_+(t_i)\left(\frac{t}{t_i}\right)^{2/3}, \quad \delta_-(t) = \delta_-(t_i)\left(\frac{t}{t_i}\right)^{-1}}$$

Note that the solutions have no dependence on k, which makes sense as dark matter has no pressure gradient. The existence of the decaying solution looks counter-intuitive because one expects perturbations grow due to gravitational attraction. The decaying solution corresponds to the case that matter in the overdense region has an initial velocity that is toward the underdense region.

The general solution is a linear combination of $\delta_+(t)$ and $\delta_-(t)$. At late time the growing solution wins, and hence perturbations grow. In a static background the growth is exponential in time. In an expanding universe, however, the damping results in a slower growth that is power law.

– Now let us consider a realistic two-component fluid consisting of dark matter and baryons. Dark matter perturbations grow at all times (particularly at $t > t_{EQ}$), while baryon perturbations grow for $k < k_J$ modes. The modes for which $k > k_J$ undergo oscillations as we discussed last time.

Defining $\delta_{DM} = \frac{\delta \rho_{DM}}{\rho_{DM}}$, $\delta_B = \frac{\delta \rho_B}{\rho_B}$ we find:

$$\ddot{\delta}_{DM} + \frac{4}{3t}\dot{\delta}_{DM} - \frac{2}{3t^2}(\delta_{DM}' + \delta_B') = 0 \qquad \delta_{DM}' \equiv \frac{\delta\rho_{DM}}{\rho_0}, \quad \delta_B' \equiv \frac{\delta\rho_B}{\rho_0}$$

$$\ddot{\delta}_B + \frac{4}{3t}\dot{\delta}_B + \frac{k^2}{3a^2_{(t)}}\delta_B - \frac{2}{3t^2}(\delta_B' + \delta_{DM}') = 0$$

Consider subhorizon modes of δ_{DM} and δ_B such that $\lambda > \lambda_J$ initially.

– $\lambda \gg \lambda_J$. The equation for δ_{DM} and δ_B are effectively the same as one can neglect $\frac{k^2}{a^2_{(t)}}$ term for δ_B. We then find:

$$\boxed{\begin{aligned} &\ddot{\delta}_{DM} + \frac{4}{3t}\dot{\delta}_{DM} - \frac{2}{3t^2}\left(\delta'_{DM} + \delta'_B\right) = 0 \\ &\ddot{\delta}_B + \frac{4}{3t}\dot{\delta}_B - \frac{2}{3t^2}\left(\delta'_B + \delta'_{DM}\right) = 0 \end{aligned}}$$

This implies that δ_{DM} and δ_B grow together as long as $k < k_J$ (or $\lambda \gg \lambda_J$). The main player in this regime is gravity.

- $\lambda < \lambda_J$. As time goes by, $\lambda \propto a^{+1} \propto t^{\frac{2}{3}}$ and $\lambda_J \propto H^{-1} \propto t$. Therefore the physical wavelength of a given mode will become shorther than the Jeans wavelength eventually. This does not change the equation for δ_{DM}. However, the term from pressure gradient becomes important for δ_B. We then have:

$$\ddot{\delta}_{DM} + \frac{4}{3t}\dot{\delta}_{DM} - \frac{2}{3t^2}\left(\delta'_{DM} + \delta'_B\right) = 0$$

$$\omega_B \equiv \frac{\rho_B}{\rho_0}$$

$$\ddot{\delta}_B + \frac{4}{[illegible]}\dot{\delta}_B + \left(\frac{k^2}{[illegible]} - \frac{2\omega_B}{[illegible]}\right)\delta_B - \frac{2}{[illegible]}\delta'_{DM} = 0$$

In the second equation k is the comoving wavenumber of a given mode, and we have used the fact that $a \propto t^{\frac{2}{3}}$ in the matter dominated phase.

We see that for $\lambda < \lambda_J$ dark matter perturbations δ_{DM} keep growing. On the other hand, baryon perturbations start to oscillate and are also subject to the influence of dark matter perturbations. This will go on until the recombination at which time baryons decouple from photons. At that point δ_B and δ_{DM} grow quickly as a result of gravitational clumping.

It is useful to make an analogy with harmonic oscillator since after all equations are the same as that of a harmonic oscillator (with some details like the expansion of the universe added).

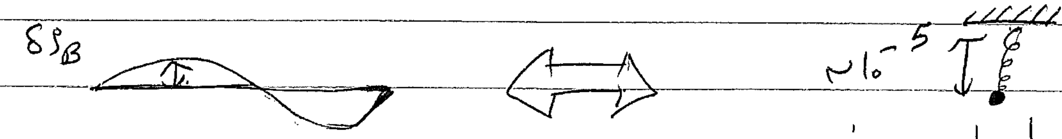
Dark matter perturbations δ_{DM} obeys the same equation as an inverted harmonic oscillator ($\omega^2 < 0$). It therefore grows at all times.

Baryon perturbations δ_B obeys the equation of an inverted oscillator as long as $k < k_J$. It grows as a function of time just like δ_{DM} in that regime. However, for $k > k_J$, it behaves like an ordinary oscillator that is "forced". This can be seen from the equation for δ_B:

$$\boxed{\ddot{\delta}_B + \frac{4}{3t}\dot{\delta}_B + \left(\frac{k^2}{3t^{4/3}} - \frac{2\omega_B}{3t^2}\right)\delta_B = \frac{2\omega_{DM}\delta_{DM}}{3t^2}} \qquad \omega_{DM} \equiv \frac{\rho_{DM}}{\rho_0}$$

δ_{DM} appears in the right-hand side of the equation and acts as an external force. An external force changes the equilibrium point of an oscillator. One can think about the evolution of δ_B as follows:

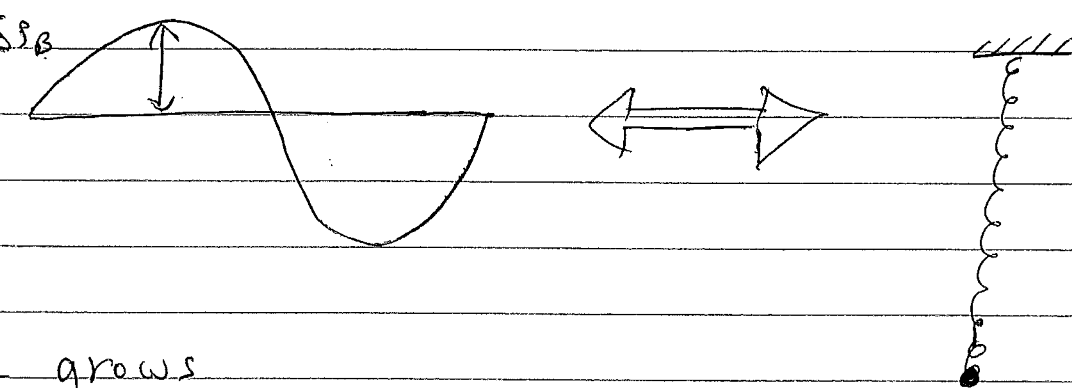
$-\lambda \sim H^{-1}$:



inverted harmonic oscillator

$\frac{\delta\rho_B}{\rho_B} \sim 10^{-5}$ (from CMB data)

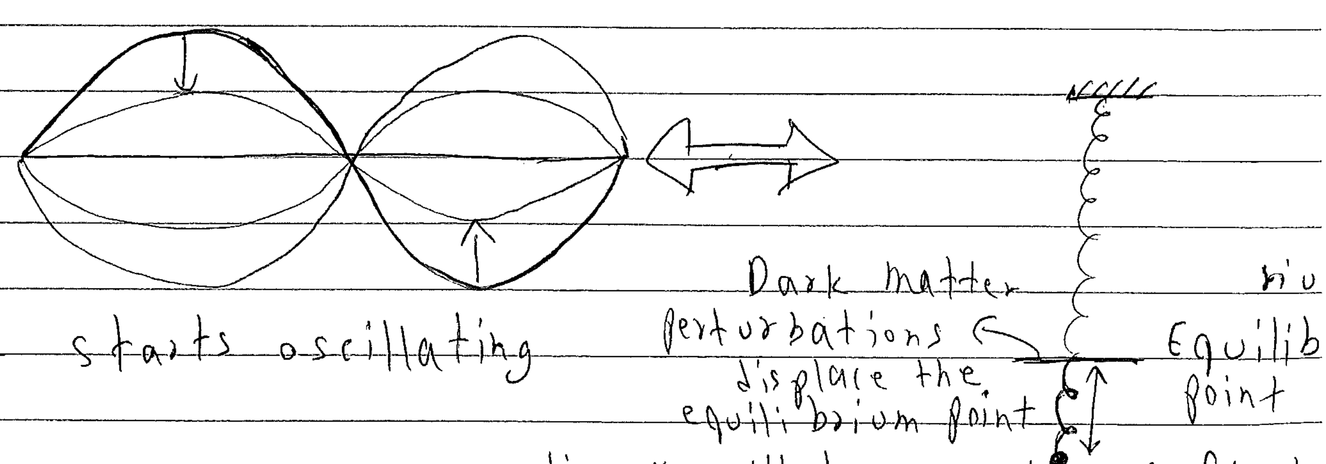
$-\lambda \sim \lambda_J$:



$\frac{\delta\rho_B}{\rho_B}$ grows

inverted harmonic oscillator

$-\lambda < \lambda_J$



$\frac{\delta\rho_B}{\rho_B}$ starts oscillating

ordinary oscillator in external field